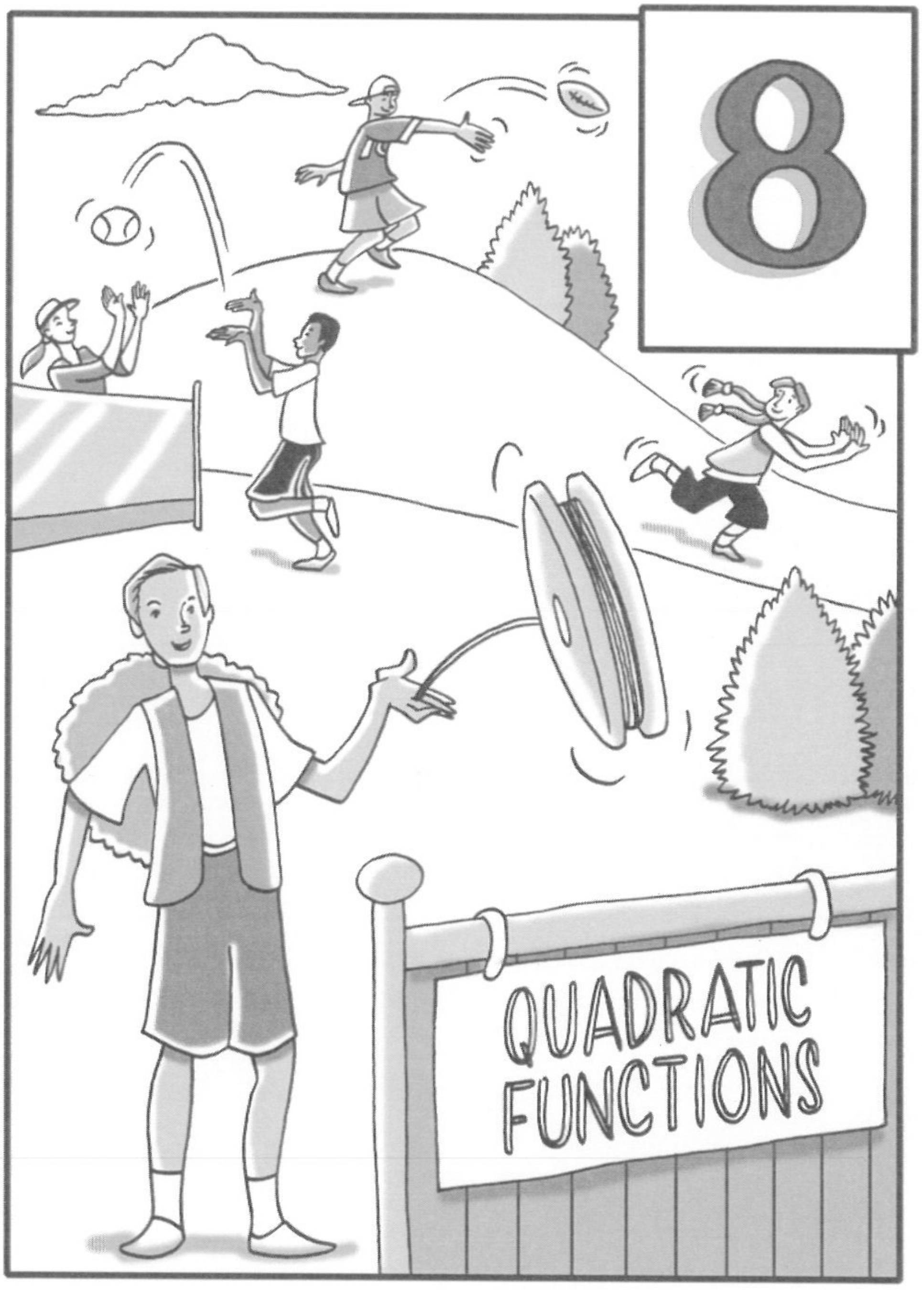

8
QUADRATIC
FUNCTIONS

CHAPTER 8 Quadratic Functions

In Chapter 2, you used a web to organize the connections you found between each of the different representations of lines. These connections enabled you to use any representation (such as a graph, rule, situation, or table) to find any of the other representations. You did the same thing in Chapter 7 for exponential functions.

In this chapter, a quadratics web will challenge you to find connections between the different representations of a quadratic function. Through this endeavor, you will learn how to rewrite quadratic equations in several forms, and how to use your graphing calculator to assist you.

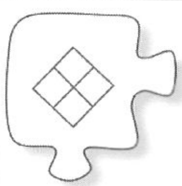
Guiding Question

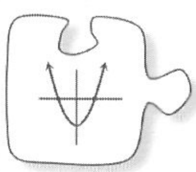
Mathematically proficient students construct viable arguments and critique the reasoning of others.

As you work through this chapter, ask yourself:

Can I explain my understanding of mathematics accurately to others?

Chapter Outline





Section 8.1 In this section, you will develop a method to change a quadratic equation written as a sum into its product form (also called its factored form). Then you will learn shortcuts for factoring some quadratics.

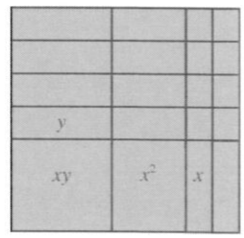
Section 8.2 Through a fun application, you will find ways to generate each representation of a quadratic function (rule, graph, table, and situation) from each of the others. You will also develop a method to find the x-intercepts of a parabola using the Zero Product Property. Then you will see another way to write the equation of a parabola, called graphing form, and use square roots to find the x-intercepts. Finally you will "complete the square" to change between standard form and graphing form of a quadratic function.

8.1.1 How can I find the product?

Introduction to Factoring Quadratic Expressions

In Chapter 3 you learned how to multiply algebraic expressions using algebra tiles and generic rectangles. This section will focus on reversing this process: How can you find a product when given a sum?

8-1. Review what you know about products and sums below.



a. Write the area of the rectangle at right as a product and as a sum. Remember that the product represents the area found by multiplying the length by the width, while the sum is the result of adding the areas inside the rectangle.

b. Use a generic rectangle to multiply $(6x-1)(3x+2)$. Write your solution as a sum.

8-2. The process of changing a sum to a product is called **factoring**. Can every expression be factored? That is, *does every sum have a product that can be represented with tiles?*

Investigate this question by building rectangles with algebra tiles for the following expressions. For each one, write the area as a sum and as a product. If you cannot build a rectangle, be prepared to convince the class that no rectangle exists (and thus the expression cannot be factored).

a. $2x^2+7x+6$

b. $6x^2+7x+2$

c. x^2+4x+1

d. $2xy+6x+y^2+3y$

8-3. Work with your team to find the sum and the product for the following generic rectangles. Are there any special strategies you discovered that can help you determine the dimensions of the rectangle? Be sure to share these strategies with your teammates.

a.

$2x$	5
$6x^2$	$15x$

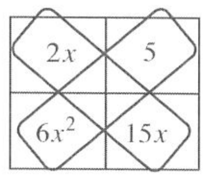
b.

$-2y$	-6
$5xy$	$15x$

c.

$-9x$	-12
$12x^2$	$16x$

8-4. While working on problem 8-3, Casey noticed a pattern with the diagonals of each generic rectangle. However, just before she shared her pattern with the rest of her team, she was called out of class! The drawing on her paper looked like the diagram below. Can you figure out what the two diagonals have in common?



8-5. Does Casey's pattern always work? Verify that her pattern works for all of the 2-by-2 generic rectangles in problem 8-3. Then describe Casey's pattern for the diagonals of a 2-by-2 generic rectangle in your Learning Log. Be sure to include an example. Title this entry "Diagonals of a Generic Rectangle" and include today's date.

MATH NOTES

METHODS AND MEANINGS

More Vocabulary for Expressions

Since algebraic expressions come in several different forms, there are special words used to help describe these expressions. For example, if the expression can be written in the form ax^2+bx+c and if a is not 0, it is called a **quadratic** expression. Review the examples of quadratic expressions below.

Examples of quadratic expressions: $x^2-15x+26$

$16m^2-25$

$12-3k^2+5k$

The way an expression is written can also be named. When an expression is written in product form, it is said to be **factored**. When factored, each of the expressions being multiplied is called a **factor**. For example, the factored form of $x^2-15x+26$ is $(x-13)(x-2)$, so $x-13$ and $x-2$ are each factors of the original expression.

Finally, if the expression is a polynomial (see Math Notes box in Lesson 3.1.2) the number of terms can help you name the polynomial. If the polynomial has one term, it is called a **monomial**, while a polynomial with two terms is called a **binomial**. If the polynomial has three terms, it is called a **trinomial**. Review the examples below.

Examples of monomials: $15y^2$ and -2

Examples of binomials: $16m-25$ and $7h^9+\frac{1}{2}$

Examples of trinomials: $12-3k^3+5k$ and $x^2-15x+26$

Review & Preview

8-6. Write the area of the rectangle at right as a sum and as a product.

$-3x$	$-6y$	12
$2x^2$	$4xy$	$-8x$

8-7. Multiply the expressions below using a generic rectangle. Then verify Casey's pattern (that the product of one diagonal equals the product of the other diagonal).

a. $(4x-1)(3x+5)$

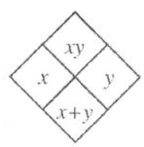
b. $(2x-7)^2$

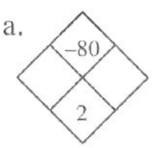
8-8. Review problem 7-108. Write the equation for the following sequences in "first term" form.

a. $500, 2000, 3500, \ldots$

b. $30, 150, 750, 3750, \ldots$

8-9. Remember that a Diamond Problem is a pattern for which the **product** of two numbers is placed on top, while the **sum** of the same two numbers is placed on bottom. (This pattern is demonstrated in the diamond at right.) Copy and complete each Diamond Problem below.



a.


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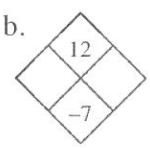

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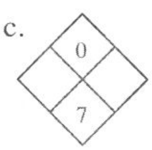

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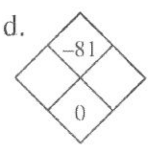

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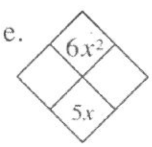

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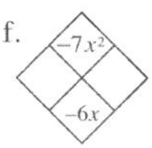


8-10. In a previous course you used the Distributive Property and common factors to change expressions written as sums into expressions written as products. For example:

Since 6 is the greatest common factor of both terms, $12x+18$ may be rewritten:

Here x is a common factor of every term, so x^2+xy+x may be rewritten:
$x^2+xy+x=x(x+y+1)$.

Use the greatest common factor to rewrite each sum as a product.

a. $4x+8$

b. $10x+25y+5$

c. $2x^2-8x$

d. $9x^2y+12x+3xy$

8-11. On graph paper, graph $y=x^2-2x-8$.

a. Name the y-intercept. What is the connection between the y-intercept and the rule $y=x^2-2x-8$?

b. Name the x-intercepts.

c. Find the lowest point of the graph, called the vertex.

8-12. Calculate the value of each expression below.

a. $5-\sqrt{36}$

b. $1+\sqrt{39}$

c. $-2-\sqrt{5}$

8.1.2 Is there a shortcut?

Factoring with Generic Rectangles

Since mathematics is often described as the study of patterns, it is not surprising that generic rectangles have many patterns. You saw one important pattern in Lesson 8.1.1 (Casey's pattern from problem 8-4). Today you will continue to use patterns while you develop a method to factor trinomial expressions.

8-13. Examine the generic rectangle shown at right.

$-35x$	14
$10x^2$	$-4x$

a. Review what you learned in Lesson 8.1.1 by writing the area of the rectangle at right as a sum and as a product.

b. Does this generic rectangle fit Casey's pattern for diagonals? Demonstrate that the product of each diagonal is equal.

8-14. FACTORING QUADRATIC EXPRESSIONS

To develop a method for factoring without algebra tiles, first model how to factor with algebra tiles, and then look for connections within a generic rectangle.

a. Using algebra tiles, factor $2x^2+5x+3$; that is, use the tiles to build a rectangle, and then write its area as a product.

b. To factor with tiles (like you did in part (a)), you need to determine how to arrange the tiles to form a rectangle. Using a generic rectangle to factor requires a different process.

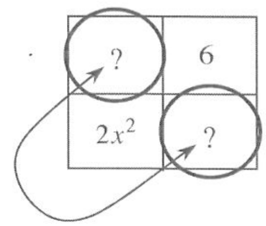
Miguel wants to use a generic rectangle to factor $3x^2+10x+8$. He knows that $3x^2$ and 8 go into the rectangle in the locations shown at right. Finish the rectangle by deciding how to place the ten x-terms. Then write the area as a product.

	8
$3x^2$	

Problem continues on next page. →

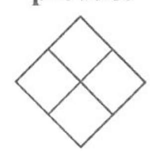
8-14. *Problem continued from previous page.*

c. Kelly wants to find a shortcut to factor $2x^2+7x+6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?



d. To complete Kelly's generic rectangle, you need two x-terms that have a sum of $7x$ and a product of $12x^2$. Create and solve a Diamond Problem that represents this situation.

product

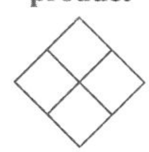


sum

e. Use your results from the Diamond Problem to complete the generic rectangle for $2x^2+7x+6$, and then write the area as a product of factors.

8-15. Factoring with a generic rectangle is especially convenient when algebra tiles are not available or when the number of necessary tiles becomes too large to manage. Using a Diamond Problem helps avoid guessing and checking, which can at times be challenging. Use the process from problem 8-14 to factor $6x^2+17x+12$. The questions below will guide your process.

product



sum

a. When given a trinomial, such as $6x^2+17x+12$, what two parts of a generic rectangle can you quickly complete?

b. How can you set up a Diamond Problem to help factor a trinomial such as $6x^2+17x+12$? What goes on the top? What goes on the bottom?

c. Solve the Diamond Problem for $6x^2+17x+12$ and complete its generic rectangle.

d. Write the area of the rectangle as a product.

8-16. Use the process you developed in problem 8-14 to factor the following quadratics, if possible. If a quadratic cannot be factored, justify your conclusion.

a. $x^2+9x+18$

b. $4x^2+17x-15$

c. $4x^2-8x+3$

d. $3x^2+5x-3$

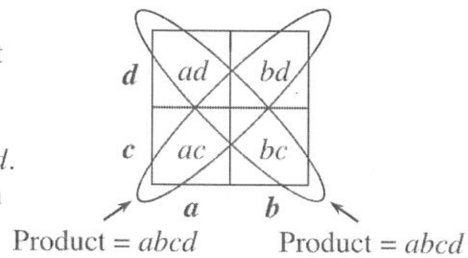
MATH NOTES

METHODS AND MEANINGS

Diagonals of Generic Rectangles

Why does Casey's pattern from problem 8-4 work? That is, why does the product of the terms in one diagonal of a 2-by-2 generic rectangle always equal the product of the terms in the other diagonal?

Examine the generic rectangle at right for $(a+b)(c+d)$. Notice that each of the resulting diagonals have a product of $abcd$. Thus, the product of the terms in the diagonals are equal.



Review & Preview

8-17. Use the process you developed in problem 8-14 to factor the following quadratic expressions, if possible.

a. $x^2-4x-12$

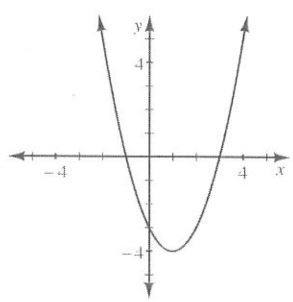
b. $4x^2+4x+1$

c. $2x^2-9x-5$

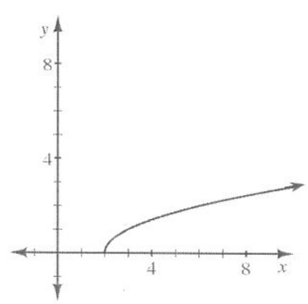
d. $3x^2+10x-8$

8-18. For each rule represented below, state the x- and y-intercepts, if possible.

a.



b.



c.

x	−5	−4	−3	−2	−1	0	1	2
y	8	4	0	−4	0	2	0	−4

d. $5x-2y=40$

8-19. Write the equation for the following two sequences in "first term" form.

a. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

b. $-7.5, -9.5, -11.5, \ldots$

8-20. The value of Bulls Eye stock has decreased 8% each year for the past several years. If in 2010 the stock was worth \$50 and that pattern continues, how much will it be worth in 2015?

8-21. Find the point of intersection for each system.

a. $y = 2x - 3$
$x + y = 15$

b. $3x = y - 2$
$6x = 4 - 2y$

8-22. Solve each equation below for the given variable, if possible.

a. $\frac{4x}{5} = \frac{x-2}{7}$

b. $-3(2b - 7) = -3b + 21 - 3b$

c. $6 - 2(c - 3) = 12$

8-23. Find the equation of the line that passes through the points (–800, 200) and (–400, 300).

8.1.3 How can I factor this?

Factoring with Special Cases

Practice your new method for factoring quadratic expressions without tiles as you consider special types of quadratic expressions.

8-24. Factor each quadratic expression below, if possible. Use a Diamond Problem and generic rectangle for each one.

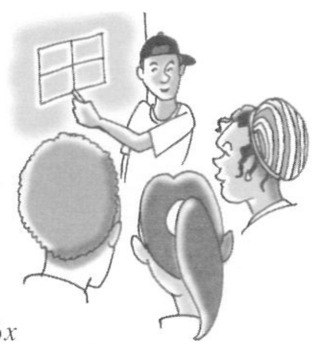
a. x^2+6x+9

b. $2x^2+5x+3$

c. x^2+5x-7

d. $3m^2+m-14$

8-25. SPECIAL CASES



Most quadratic expressions are written in the form ax^2+bx+c. But what if a term is missing? Or what if the terms are in a different order? Consider these questions while you factor the expressions below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

a. $9x^2-4$

b. $12x^2-16x$

c. $3+8k^2-10k$

d. $40-100m$

8-26. Now turn your attention to the quadratic expression below. Use a generic rectangle and Diamond Problem to factor this expression. Compare your answer with your teammates' answers. Is there more than one possible answer?

$$4x^2-10x-6$$

8-27. The multiplication table below has factors along the top row and left column. Their product is where the row and column intersect. With your team, complete the table with all of the factors and products.

Multiply	$x-2$	
$x+7$		
	$3x^2-5x-2$	$6x^2+5x+1$

8-28. In your Learning Log, explain how to factor a quadratic expression. Be sure to offer examples to demonstrate your understanding. Include an explanation of how to deal with special cases, such as when a term is missing or when the terms are not in standard order. Title this entry "Factoring Quadratics" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Standard Form of a Quadratic Expression

A quadratic expression in the form ax^2+bx+c is said to be in **standard form**. Notice that the terms are in order from greatest exponent to least.

Examples of quadratic expressions in standard form: $3m^2+m-1$, x^2-9, and $3x^2+5x$. Notice that in the second example, $b=0$, while in the third example, $c=0$.

Review & Preview

8-29. At 3:25 p.m., two trains left Kalamazoo, Michigan. One train traveled westward at a constant rate of 82 miles per hour, while the other traveled eastward at a constant rate of 66 miles per hour. If they are now 111 miles apart, what time is it now? Write and solve an equation (or system of equations) to answer this question.

8-30. Remember that a square is a rectangle with four equal sides.

a. If a square has an area of 81 square units, how long is each side?

b. Find the length of the side of a square with area 225 square units.

c. Find the length of the side of a square with area 10 square units.

d. Find the area of a square with side 11 units.

8-31. Factor the following quadratic expressions, if possible.

a. $k^2 - 12k + 20$ b. $6x^2 + 17x - 14$

c. $x^2 - 8x + 16$ d. $9m^2 - 1$

e. Parts (a) through (e) are trinomials while part (d) is a binomial, yet they are all quadratics. What makes each of them a quadratic?

8-32. Change each expression into radical form and then give the value. No calculators should be necessary.

a. $125^{2/3}$ b. $16^{1/2}$ c. $16^{-1/2}$ d. $(\frac{1}{81})^{1/4}$

8-33 Solve each equation below for x. Check each solution.

a. $2x - 10 = 0$ b. $x + 6 = 0$ c. $(2x - 10)(x + 6) = 0$

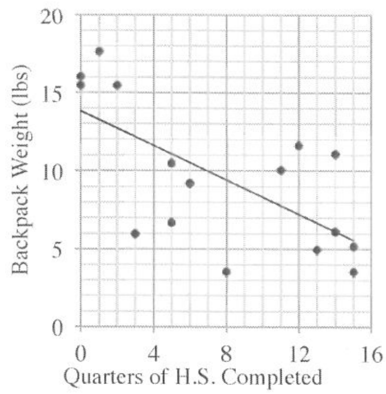
d. $4x + 1 = 0$ e. $x - 8 = 0$ f. $(4x + 1)(x - 8) = 0$

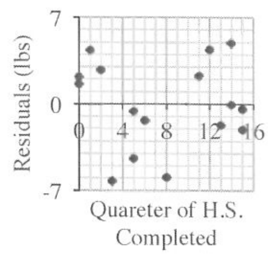
8-34. Do the freshmen really have the largest backpacks, or is that just high school legend stuff? Delenn was able to weigh a random sample of student backpacks throughout the school year. She also recorded the number of quarters of high school completed by the student who owns the bag. Using spreadsheet software, Delenn found the following:

Qtrs Completed	Backpack Weight (lbs)
0	16.03
0	15.47
8	3.52
12	11.62
1	17.66
5	6.67
5	10.48
13	4.96
2	15.47
6	9.21
14	6.10
14	11.10
3	5.96
11	10.06
15	3.54
15	5.18

LSRL $y = 13.84 - 0.55x$

$r = -0.66$





a. Interpret the slope of the least squares regression line in the context of this study.

b. Calculate and interpret R-squared in context.

c. What is the residual with the greatest magnitude and what point does it belong to?

d. Using the LSRL model, estimate the weight of a backpack for a student who has completed 10 quarters of high school. Use appropriate precision in your answer.

e. Is a linear model the best choice for predicting backpack weight in this study? Support your answer.

8.1.4 Can it still be factored?

Factoring Completely

There are several ways to write the number 12 as a product of factors. For example, 12 can be rewritten as $3 \cdot 4$, as $2 \cdot 6$, as $1 \cdot 12$, or as $2 \cdot 2 \cdot 3$. While each of these products is accurate, only $2 \cdot 2 \cdot 3$ is considered to be **factored completely**, since the factors are prime and cannot be factored themselves.

During this lesson you will learn more about what it means for a quadratic expression to be factored completely.

8-35. Review what you have learned by factoring the following expressions, if possible.

a. $9x^2 - 12x + 4$

b. $81m^2 - 1$

c. $28 + x^2 - 11x$

d. $3n^2 + 9n + 6$

8-36. Compare your solutions for problem 8-35 with the rest of your class.

a. Is there more than one factored form of $3n^2 + 9n + 6$? Why or why not?

b. Why does $3n^2 + 9n + 6$ have more than one factored form while the other quadratics in problem 8-35 only have one possible answer? Look for clues in the original expression ($3n^2 + 9n + 6$) and in the different factored forms.

c. *Without factoring*, predict which quadratic expressions below may have more than one factored form. Be prepared to defend your choice to the rest of the class.

i. $12t^2 - 10t + 2$

ii. $5p^2 - 23p - 10$

iii. $10x^2 + 25x - 15$

iv. $3k^2 + 7k - 6$

8-37. FACTORING COMPLETELY

In part (c) of problem 8-36, you should have noticed that each term in $12t^2 - 10t + 2$ is divisible by 2. That is, it has a **common factor** of 2.

a. An expression is considered **completely factored** if none of the factors can be factored any more. Often it is easiest to remove common factors first, before factoring with a generic rectangle. Rewrite this expression $10x^2 + 25x - 15$ with the common factor factored out.

b. Your result in part (a) is not completely factored if either factor can be factored. Factor $10x^2 + 25x - 15$ completely.

8-38. Factor each of the following expressions as completely as possible.

a. $5x^2 + 15x - 20$

b. $3x^3 - 6x^2 - 45x$

c. $2x^2 - 50$

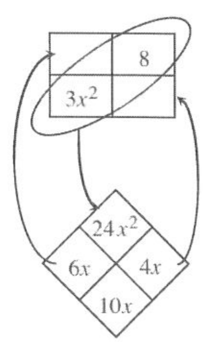
d. $x^2y - 3xy - 10y$

MATH NOTES

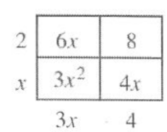
METHODS AND MEANINGS

Factoring Quadratic Expressions

Review the process of factoring quadratic expressions developed in problem 8-13 and outlined below. This example demonstrates how to factor $3x^2 + 10x + 8$.



1. Place the x^2 and constant terms of the quadratic expression in opposite corners of a generic rectangle. Determine the sum and product of the two remaining corners: The sum is simply the x-term of the quadratic expression, while the product is equal to the product of the x^2 and constant terms.
2. Place this sum and product into a Diamond Problem and solve it.
3. Place the solutions from the Diamond Problem into the generic rectangle and find the dimensions of the generic rectangle.



4. Write your answer as a product: $(3x + 4)(x + 2)$.

Review & Preview

8-39. Factor the quadratic expressions below. If the quadratic is not factorable, explain why not.

a. $2x^2+3x-5$

b. x^2-x-6

c. $3x^2+13x+4$

d. $2x^2+5x+7$

8-40. A sequence starts $-3, 1, 5, 9\ldots$

a. If you wanted to find the 50^{th} term of the sequence, would an explicit equation or a recursive equation be more useful?

b. Write the equation in "first term" form as you did in problem 7-108.

c. What is the 50^{th} term of the sequence?

d. Write the explicit equation for the sequence $3,\ 2\frac{2}{3},\ 2\frac{1}{3},\ 2,\ 1\frac{2}{3},\ \ldots$ in "first term" form.

8-41. As Jhalil and Joman practice for the SAT, their scores on practice tests rise. Jhalil's current score is 850, and it is rising by 10 points per week. On the other hand, Joman's current score is 570 and is growing by 50 points per week.

a. When will Joman's score catch up to Jhalil's?

b. If the SAT test is in 12 weeks, who will score highest?

8-42. Mary says that you can find an x-intercept by substituting 0 for x, while Michelle says that you need to substitute 0 for y.

a. Who, if anyone, is correct and why?

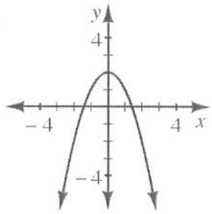
b. Use the correct approach to find the x-intercept of $-4x+5y=16$.

8-43. Find three consecutive integers whose sum is 138 by writing and solving an equation.

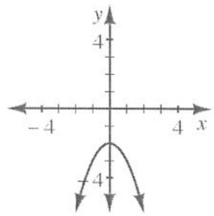
8-44. Match each rule below with its corresponding graph. Can you do this without making any tables? Explain your selections.

a. $y=-x^2-2$ b. $y=x^2-2$ c. $y=-x^2+2$

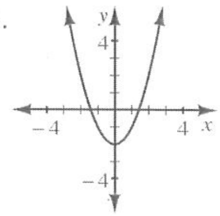
1.



2.



3.



8.1.5 Is there a shortcut?

Factoring Shortcuts

Are there any types of quadratic expressions that you can factor quickly without using a generic rectangle? If so, what do these quadratics look like and how can you recognize them? Today your team will examine the factored forms of several different quadratic expressions and look for patterns and shortcuts for factoring them.

8-45. SPECIAL QUADRATICS

Your team will be assigned several of the quadratic expressions below to factor (if possible). Look for similarities and differences among the expressions and their corresponding factored forms. Be prepared to share your factored results with the class. Then work as a class to sort them into groups based on the patterns you find in their factored forms.

a. x^2-49 b. $x^2+2x-24$ c. $x^2-10x+25$

d. $9x^2+12x+4$ e. $5x^2-4x-1$ f. $4x^2-25$

g. x^2-6x+9 h. x^2-36 i. $7x^2-20x-3$

j. $4x^2+20x+25$ k. x^2+4 l. $9x^2-1$

8-46. Which of the following quadratic expressions fit the patterns you found in problem 8-45? Factor each of the following expressions using your new shortcuts, if possible.

a. $25x^2-1$ b. $x^2-5x-36$ c. $x^2+8x+16$

d. $9x^2-12x+4$ e. $9x^2+4$ f. $9x^2-100$

8-47. Special quadratic expressions, like $9x^2-100$ in part (f) of problem 8-46, can be factored quickly once you discover the pattern. But why do the patterns you found in problem 8-45 work?

a. A quadratic expression in the form $a^2x^2-b^2$ is called a **difference of squares**. Use a generic rectangle to prove that $a^2x^2-b^2=(ax-b)(ax+b)$. Be ready to share your work with the class.

b. A quadratic expression in the form $a^2x^2+2abx+b^2$ is called a **perfect square trinomial**. Use a generic rectangle to prove that $a^2x^2+2abx+b^2=(ax+b)^2$. Be ready to share your work with the class.

8-48. In your Learning Log, describe how to factor a difference of squares and a perfect square trinomial. Be sure to include an example of each type. Title this entry "Factoring Shortcuts" and include today's date.

Review & Preview

8-49. Factor each polynomial.

a. x^2-64 b. y^2-6y+9

c. $4x^2+4x+1$ d. $5x^2-45$

8-50. Simplify each expression below. Your answer should contain no parentheses and no negative exponents.

a. $(-\frac{2}{3}x^5y^{1/3})^0$ b. $(25^{1/2}x^5)(4x^{-6})$

c. $5t^{-3}$ d. $(\frac{x^7y^3}{x})^{1/3}$

8-51. Solve each of the following systems of equations algebraically. Then confirm your solutions by graphing.

a. $y = 4x + 5$
$y = -2x - 13$

b. $2x + y = 9$
$y = -x + 4$

8-52. Consider the sequence 4, 8, …

a. If the sequence is arithmetic, write the first 4 terms and an equation in "first term" form for $t(n)$.

b. If the sequence is geometric, write the first 4 terms and an equation in "first term" form for $t(n)$.

c. Create another sequence that is neither arithmetic nor geometric and still starts with 4, 8, ….

8-53. Solve the following equations for x.

a. $4x - 6y = 20$

b. $\frac{1}{2}(x - 6) = 9$

c. $\frac{4}{5} + \frac{18}{x} = 8$

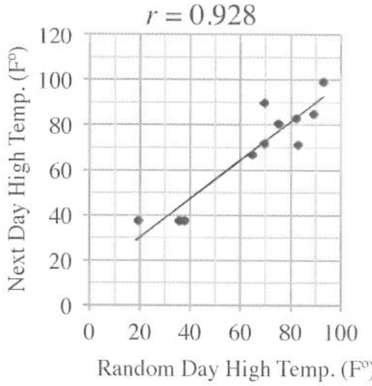
d. $2 + |2x - 3| = 5$

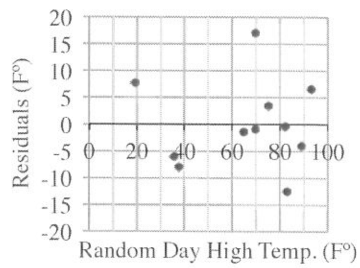
8-54. Mitchell likes to study the weather. He is fascinated by the sophistication of the computer models used to make weather predictions. Mitchell wonders if he can make his own model to predict the next day's high temperature in his area based only on today's high temperature. He selects 11 days at random and gets the temperatures from the Internet. The results from his computer spreadsheet follow.

Random Day (F°)	Next Day (F°)
93.2	99
69.8	71.6
82.9	71.1
82.4	82.9
19.4	37.4
69.8	89.6
35.6	37.4
89.1	84.9
37.9	37.4
75.2	80.6
64.9	66.9

LSRL $y = 13.17 + 0.85x$

$r = 0.928$





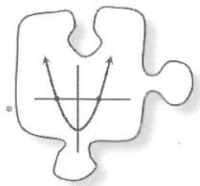
a. Write a few sentences that describe the association. Remember to include interpretations of slope and R^2.

b. Use the graph to estimate the largest residual. To what point does it belong?

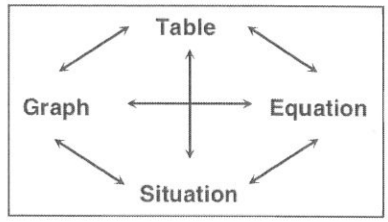
c. Using the LSRL model, estimate tomorrow's high temperature based on today's high temperature of 55 degrees in Mitchell's area. Use appropriate precision.

d. Consider the upper and lower bounds of the prediction Mitchell made in part (c) above. Is Mitchell's model ready to replace the complex models of the professional meteorologists? Support your answer.

8.2.1 What is the connection?



Multiple Representations for Quadratic Functions



In Chapter 2 you completed a web for the different representations of the family of linear functions and in Chapter 7 you did the same thing for the family of exponential functions. You discovered shortcuts to help you move from one representation to another.

Today you will explore the connections between the different representations for quadratic functions. As you work, keep in mind the following questions:

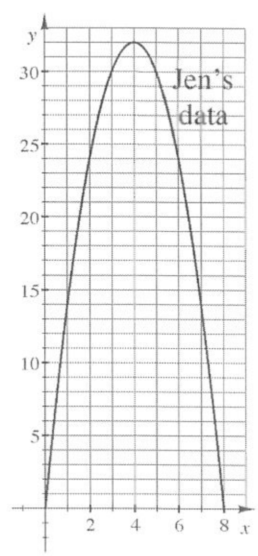
What representations are you using?

What is the connection between the various representations?

What do you know about a parabola?

8-55. WATER BALLOON CONTEST

Every year Newtown High School holds a water balloon competition during halftime of their homecoming game. Each contestant uses a catapult to launch a water balloon from the ground on the football field. This year you are the judge! You must decide which contestants win the prizes for *Longest Distance* and *Highest Launch*. Fortunately, you have a computer that will collect data for each throw. The computer uses x to represent horizontal distance in yards from the goal line and y to represent the height in yards.



The announcer shouts, "Maggie Nanimos, you're up first!" She runs down and places her catapult at the 3-yard line. After Maggie's launch, the computer reports that the balloon traveled along the parabola represented by the quadratic equation $y = -x^2 + 17x - 42$.

Then you hear, "Jen Erus, you're next!" Jen runs down to the field, places her catapult at the goal line, and releases the balloon. The tracking computer reports the path of the balloon with the graph at right.

Problem continues on next page. →

8-55. *Problem continued from previous page.*

The third contestant, Imp Ecable, accidentally launches the balloon before you are ready. The balloon launches, you hear a roar from the crowd, turn around, and…SPLAT! The balloon soaks you and your computer! You only have time to write down the following partial information about the balloon's path before your computer fizzles:

x (yards)	2	3	4	5	6	7	8	9
y (yards)	0	9	16	21	24	25	24	21

Finally, the announcer calls for the last contestant, Al Truistic. With your computer broken, you decide to record the balloon's height and distance by hand. Al releases the balloon from the 10 yard line. The balloon reaches a height of 27 yards and lands at the 16 yard line.

a. Obtain the Lesson 8.2.1A Resource Page from your teacher. For each contestant, create a table and graph using the information provided for each toss. Determine which of these contestants should win the *Longest Distance* and *Highest Throw* contests.

b. Find the x-intercepts of each parabola. What information do the x-intercepts tell you about each balloon toss?

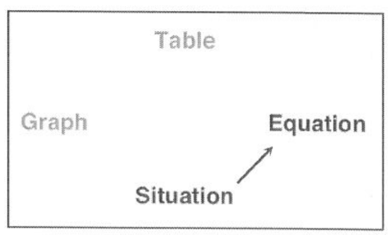
c. Find the vertex of each parabola. What information does the vertex tell you about each balloon throw?

d. What is the domain and range of Maggie's parabola?

e. Where is the line of symmetry for Maggie's parabola?

8-56. SITUATION TO RULE

Write an explicit equation from the situation described by the tile pattern below.



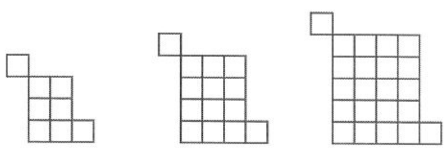


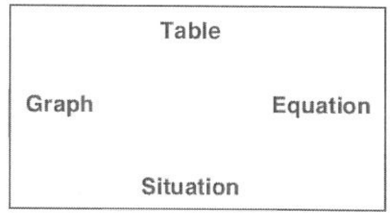
Figure 1 Figure 2 Figure 3

a. Write a rule to represent the number of tiles in Figure x.

b. Is the rule from part (a) quadratic? Explain how you know.

8-57. LEARNING LOG

Today you have explored the four different representations of quadratic functions: table, graph, equation, and a description of a physical situation involving motion. Draw the representations of the web as shown at right in your Learning Log. Label this entry "Quadratic Web" and date it.



Draw in arrows showing the connections that you currently know how to make between different representations. Be prepared to justify a connection for the class.

Review & Preview

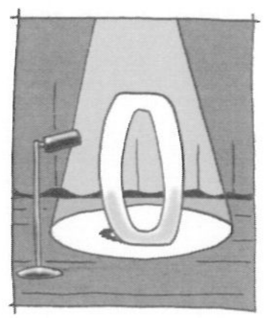
8-58. Graph $y = x^2 - 8x + 7$ and label its vertex, x-intercepts, and y-intercepts.

8-59. What is special about the number zero? Think about this as you answer the questions below.

a. Find each sum:

$0 + 3 =$ $\quad$ $-7 + 0 =$ $\quad$ $0 + 6 =$ $\quad$ $0 + (-2) =$

b. What is special about adding zero? Write a sentence that begins, "When you add zero to a number, …"



c. Julia is thinking of two numbers a and b. When she adds them together, she gets a sum of b. Does that tell you anything about either of Julia's numbers?

d. Find each product:

$3 \cdot 0 =$ $\quad$ $(-7) \cdot 0 =$ $\quad$ $0 \cdot 6 =$ $\quad$ $0 \cdot (-2) =$

e. What is special about multiplying by zero? Write a sentence that begins, "When you multiply a number by zero, …"

8-60. Based on the tables below, say as much as you can about the x- and y-intercepts of the corresponding graphs.

a.

x	y
2	0
0	18
–4	0
–1	–8
6	22
3	0

b.

x	y
7	–4
3	0
10	8
0	–3
8	0
–7	–1

c.

x	y
0	–4
–5	11
3	–2
1	0
13	27
–6	14

8-61. In speed golfing an athlete's score is determined by adding the number of strokes to complete a course to the minutes required to finish. For example 90 strokes in 51 minutes would be a score of 141. The lower the score, the better. Diego wants to see if there is a relationship between the time, t, it takes for him to complete a speed golfing match and the number of strokes, s, he takes in the same match. If so, perhaps focusing on running faster will also reduce the number of strokes.

Time, t	56	92	56	58	45	50
Strokes, s	86	90	80	91	77	86

a. Create a scatterplot with pencil and paper. Determine Diego's best score and circle the point representing Diego's best total score.

b. Discuss what you can about the association from observation of the scatterplot.

c. Diego recalls that he was suffering from seasonal allergies that slowed his running on a particular course. Cross-out that point. Then use your intuition and draw a line of best fit from the remaining points.

d. Estimate the slope of your trend line and interpret it in the context of the problem.

e. Should Diego train to reduce his time so that he sees an increase in his golf score?

8-62. Solve the following systems of equations using any method. Check your solution if possible.

a. $6x - 2y = 10$
$3x - y = 2$

b. $x - 3y = 1$
$y = 16 - 2x$

8-63. The "≤" symbol represents "less than or equal to," while the "<" symbol represents "less than."

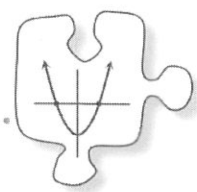
a. Similarly, translate "≥" and ">."

b. How can you write an expression that states that 5 is greater than 3?

c. Write another expression that states that x is less than or equal to 9.

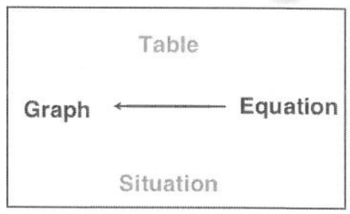
d. Translate the expression $-2 < 7$ into words.

8.2.2 How are quadratic rules and graphs connected?



Zero Product Property

You already know a lot about quadratic functions and you have made several connections between the different representations on the quadratic web. Today you are going to develop a method to sketch a parabola from its equation without a table.



8-64. WHAT DO YOU NEED TO SKETCH A PARABOLA?

How many points do you need in order to sketch a parabola? One? Ten? Fifty? Think about this as you answer the questions below. (Note: A sketch does not need to be exact. The parabola merely needs to be reasonably placed with important points clearly labeled.)

a. Can you sketch a parabola if you only know where its y-intercept is? For example, if the y-intercept of a parabola is at $(0, -15)$, can you sketch its graph? Why or why not?

b. What about the two x-intercepts of the parabola? If you only know where the x-intercepts are, can you draw the parabola? For example, if the x-intercepts are at $(-3, 0)$ and $(5, 0)$, can you predict the path of the parabola?

c. Can you sketch a parabola with only its x-intercepts and y-intercept? To test this idea, sketch the graph of a parabola for the quadratic equation $y = x^2 - 2x - 15$ with x-intercepts $(-3, 0)$ and $(5, 0)$ and y-intercept $(0, -15)$.

d. Where is the line of symmetry?

8-65. In problem 8-64, you learned that if you can find the intercepts of a parabola from a rule, then you can sketch the graph without a table.

a. What is true about the value of y for all x-intercepts? What is true about the value of x for all y-intercepts? Review your knowledge of intercepts and describe it here.

b. If $x = 0$ at the y-intercept, find the y-intercept of the graph of $y = 2x^2 + 5x - 12$.

c. Since the x-intercept occurs when $y = 0$, write the equation that you would need to solve to find the x-intercepts for the graph of $y = 2x^2 + 5x - 12$.

d. The **roots** or **zeros** of a quadratic expression are the values of x that make the value of the quadratic equal to zero. An x-intercept of a quadratic function is a root. At this point, can you solve $2x^2 + 5x - 12 = 0$ for x? Explain why or why not.

8-66. ZERO PRODUCT PROPERTY

The equation you wrote in part (c) of problem 8-65 is called a **quadratic equation**. To solve it, you need to examine what you know about zero. Study the special properties of zero below.

Nathan, Sonia, and Gaston are playing a game where Nathan and Sonia each think of a number and then give Gaston a clue about their numbers. Using the clue, Gaston must tell them everything that he knows about their numbers.

a. Nathan and Sonia's first clue for Gaston is that when you multiply their numbers together, the result is zero. What conclusion can Gaston make?

b. Disappointed that Gaston came so close to figuring out their numbers, Nathan and Sonia invite Nadia over to make things harder. Nathan, Sonia, and Nadia all think of secret numbers. This time Gaston is told that when their *three* secret numbers are multiplied together, the answer is zero. What can Gaston conclude this time?

c. Does it matter how many numbers are multiplied? If the product is zero, what do you know about one of the numbers? This property is called the **Zero Product Property**. With the class, write a description of this property in your Learning Log. Title this entry "Zero Product Property" and include today's date.

8-67. Let's investigate how can you use the Zero Product Property to help you solve the quadratic equation $2x^2+5x-12$ from part (d) of problem 8-65.

a. Examine the quadratic equation. Is there a product that equals zero? If not, how can you rewrite the quadratic expression as a product?

b. Now that the equation is written as a product of factors equaling zero, you can use the Zero Product Property to solve it. Since you know that one of the factors must be zero, you can set up two simpler equations to help you solve for x. Use one factor at a time and determine what x-value makes it equal to zero.

c. What do these solutions represent? What do they tell you?

d. You now know the roots (also called the zeros) of $2x^2+5x-12$. Use the roots to find the x-intercepts of the graph of the parabola $2x^2+5x-12$. Then sketch a graph of the parabola.

8-68. Can you make a sketch of a parabola from the two x-intercepts and the *vertex*?

a. Find the x-intercepts of the parabola $y=x^2-2x-8$. Then find the vertex and make a sketch.

b. Which was easier, using the vertex or using the y-intercept to draw the parabola?

MATH NOTES

METHODS AND MEANINGS

Zero Product Property

When the product of two or more numbers is zero, one of those numbers must be zero. This is known as the **Zero Product Property**. If the two numbers are represented by a and b, this property can be written as follows:

If a and b are two numbers where $a \cdot b=0$, then $a=0$ or $b=0$.

For example, if $(2x-3)(x+5)=0$, then $2x-3=0$ or $x+5=0$. Solving yields the solutions $x=\frac{3}{2}$ or $x=-5$. This property helps you solve quadratic equations when the equation can be written as a product of factors.

Review & Preview

8-69. Use a similar process as you did in problems 8-65 and 8-67 to sketch the parabola for x^2+x-6 by using its intercepts.

8-70. Compare the two equations below.

$$(x+2)(x-1)=0 \text{ and } (x+2)+(x-1)=0$$

a. How are the equations different? b. Solve both equations.

8-71. For each equation below, solve for x.

a. $(x-2)(x+8)=0$

b. $(3x-9)(x-1)=0$

c. $(x+10)(2x-5)=0$

d. $(x-7)^2=0$

8-72. Examine the system of equations below.

$$5x-2y=4$$
$$x=0$$

a. Before solving this system, Danielle noticed that the point of intersection is also the y-intercept of $5x-2y=4$. Explain how she knows this.

b. Find the point of intersection of the two rules above.

8-73. The x-intercepts of the graph of $y=2x^2-16x+30$ are $(3,0)$ and $(5,0)$.

a. What is the x-coordinate of the vertex? How do you know?

b. Use your answer to part (a) above to find the y-coordinate of the vertex. Then write the vertex as a point (x, y).

8-74. Factor each quadratic below completely.

a. $2x^2-2x-4$

b. $4x^2-24x+36$

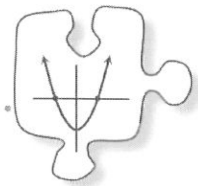
8-75. Rewrite the following expressions using fractional exponents.

a. $(\sqrt{3x})^3$

b. $\sqrt[x]{81}$

c. $(\sqrt[3]{17})^x$

8.2.3 How else can I find the roots?



More Ways to Find the x-Intercepts

In Lesson 8.2.2, you developed a method for finding the x-intercepts of a parabola in the form $y=ax^2+bx+c$ by evaluating for $y=0$. Today you will use your graphing calculator and a different form of quadratic equations to find the x-intercepts.

8-76. Review what you learned in Lesson 8.2.2 by sketching the graph of $y=2x^2+8x+6$ without a table. Specifically, find the x-intercepts and the vertex of the parabola and sketch its graph.

8-77. Use the Zero Product Property to find the roots of $x^2-3x-7=y$.

a. What happened? What does this result tell you about the roots?

b. Use your graphing calculator to display the graph of $y=x^2-3x-7$. Did the graph confirm your answer to part (b)? Estimate the roots using the graph.

c. How could you estimate the roots by looking at the table on your graphing calculator?

8-78. Estimate the x-intercepts of the following parabolas by using the graph and table on your graphing calculator.

a. $-x^2+3x+6=y$

b. $f(x)=4x^2+16x+16$

c. $y=x^2+5x+1$

d. $y=x^2-5x+9$

8-79. The following functions are also quadratic functions.

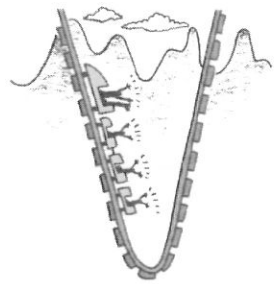
i. $y=3(x-2)^2-5$ *ii.* $f(x)=-2(x+4)^2+3$

a. Verify that both equations are quadratic by rewriting them in standard quadratic form. Remember the Order of Operations! How could you verify on your calculator that you rewrote the functions correctly?

b. Use the graph and table on your graphing calculator to find the vertex of each function. Compare the vertex to its equation. What do you notice?

c. Without graphing, can you predict the vertex of $y=\frac{1}{2}(x+3)^2+2$? Check your conjecture by making a graph with your calculator.

8-80. The function $y=(x+1)^2-16$ is quadratic. Let's explore how to draw a sketch of the parabola without making a table first.



a. Rewrite the equation in standard form, and use the Zero Product Property to find the x-intercepts and the vertex. Make a sketch of the parabola.

b. Find the x-intercepts of $y=(x+1)^2-16$ by evaluating for $y=0$. What is the vertex?

c. Obtain the Lesson 8.2.3 Resource Page and justify each step in evaluating the equation in part (b) for $y=0$.

d. Sometimes a quadratic function in $y=a(x-h)^2+k$ form is called **graphing form**. Why is this name appropriate?

e. What are the advantages of the graphing form of the equation in part (b), compared to the Zero Product Property in part (a), when finding the x-intercept and vertex?

8-81. Find the exact x-intercepts and vertex of $y=(x+2)^2-3$. Make a sketch, then check your sketch with your graphing calculator.

8-82. In your Learning Log, describe the different strategies for finding the x-intercepts in a quadratic equation. Title this entry "Strategies for Finding x-Intercepts" and include today's date.

Review & Preview

8-83. Use the Zero Product Property to find the roots of the polynomials below.

a. $3x^2 - 7x + 4$ b. $x^2 + 6x$ c. $(x+5)(-2x+3)$

8-84. Jamie was given the problem, "Find the result when the factors of $65x^2 + 212x - 133$ are multiplied together." Before she could answer, her sister, Lauren, said, "I know the answer without factoring or multiplying!" What was Lauren's answer and how did she know?

8-85. Solve the equations below for x. Check your solutions.

a. $(6x-18)(3x+2) = 0$ b. $x^2 - 7x + 10 = 0$

c. $2x^2 + 2x - 12 = 0$ d. $4x^2 - 1 = 0$

8-86. Sketch each parabola below with the given information.

a. A parabola with x-intercepts (2, 0) and (7, 0) and y-intercept (0, –8).

b. A parabola with exactly one x-intercept at (–1, 0) and y-intercept (0, 3).

c. The parabola represented by the equation $y = (x+5)(x-1)$.

8-87. Review the meanings of the inequality symbols in the box at right. Then decide if the statements below are true or false.

$<$ less than
$\le$ less than or equal to
$>$ greater than
$\ge$ greater than or equal to

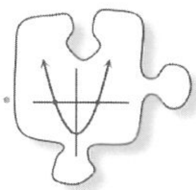
a. $5 < 7$ b. $-2 \ge 9$ c. $0 \le 0$

d. $-5 > -10$ e. $16 \le -16$ f. $1 > 1$

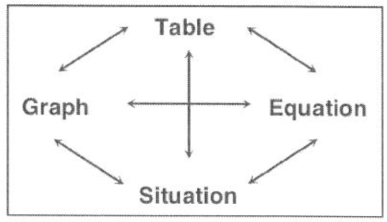
8-88. Calculate the value of each expression below using a scientific calculator.

a. $\frac{-10+\sqrt{25}}{5}$ b. $\frac{8+\sqrt{40}}{3 \cdot 3}$ c. $\frac{8+\sqrt{3^2+2 \cdot 3+1}}{-4}$

8.2.4 What is the connection?



Completing the Quadratic Web



In just three lessons you have almost completed the quadratic web. Revisit the web posted in your classroom. What connections, if any, still need to be made?

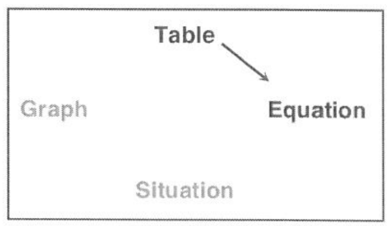
Today you will focus on how to get a quadratic equation from a table, graph, and a situation. As you work, ask yourself the following questions:

Which representation am I given?
Which representation am I looking for?

How can I reverse this process?

Is there another way?

8-89. TABLE TO RULE



You know how to make a table for a quadratic rule, but how can you write an equation when given the table? Examine this new connection that requires you to reverse your understanding of the Zero Product Property as you find a rule for each table below. What clues in the tables helped you find the rule? How can you check your equation using your graphing calculator?

a.

x	–4	–3	–2	–1	0	1	2	3	4
y	6	0	–4	–6	–6	–4	0	6	14

b.

x	–6	–5	–4	–3	–2	–1	0	1	2	3	4
y	7	0	–5	–8	–9	–8	–5	0	7	16	27

8-90. QUALITY CONTROL, Part One

Congratulations! With your promotion, you are now the Quality Assurance Representative of the Function Factory. Your job is to make sure your clients are happy. Whenever a client writes to the company, you must reply with clear directions that will solve his or her problem.

Your boss has provided graphing technology and a team of fellow employees to help you fulfill your job description.

Your Task:

1. Carefully read the complaints below. Use your graphing calculator to study each situation. Work with your team to resolve each situation.
2. Write each customer a friendly response that offers a solution to his or her problem. Remember that the customers are not parabola experts! Do not assume that they know anything about parabolas.

A

Dear Ms. Quadratic,

I followed all of the directions given in your brochure about how to order a parabola. I tried to order a parabola that passed through the points (1, 0) and (–6, 0), only to have you send me the wrong one!

Please tell me how to order the correct parabola. Your immediate reply is appreciated.

Perturbed in Pennsylvania

B

Dear Ms. Quadratic,

I am a very dissatisfied customer. I want a parabola that hits the x-axis only <u>once</u> at (5, 0), yet I see NO mention of this type of parabola in your pamphlet. Your company mission statement assures me that "my needs will be met no matter what." How should I order my special parabola?

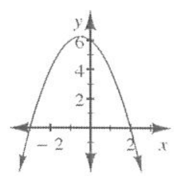
Sincerely,

Troubled in Texas

C

Dear Ms. Quadratic,

Please help! I have searched through your entire brochure and did not see a parabola that would fit my needs. All I want is a parabola that looks like this:



Every time I order an equation to give me this parabola you always send me a different one! I refuse to pay for any parabola but the one shown above. Please tell me how I should find the equation of this parabola or I will take my business elsewhere!

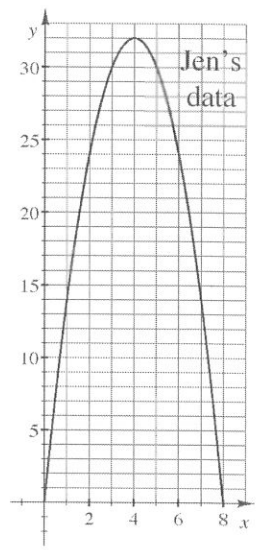
Thank you,

Agitated in Alaska

8-91. MORE CONNECTIONS

A journalist from the school newspaper wants to publish the results from the water-balloon contest. She wants a rule for each toss so that she can program her computer to create a graph for her article. You already have a rule for the toss made by Maggie from problem 8-55.

a. The journalist knows that the factored form of an equation is $y = a(x+b)(x+c)$. What do b and c represent in the factored form?



b. Examine the graph at right that represents the height of Jen's toss. The journalist found the rule for this parabola was $y = a(x-0)(x-8)$, but did not know what value a was. Explain to the journalist how she could find the value of a.

c. If you have not already done so, find the equation for Jen's toss. Write it in factored form and standard form, and check by graphing it on your calculator.

d. Al released his balloon from the 10-yard line, and it landed at the 16-yard line. If the ball reached a height of 27 yards, what equation represents the path of his toss?

e. Remember Imp's water-balloon toss? Since the water balloon was thrown on the computer, you were given only a table of data, shown again below. Find a rule that represents the height of Imp's balloon as it traveled through the air.

x (yards)	2	3	4	5	6	7	8	9
y (yards)	0	9	16	21	24	25	24	21

MATH NOTES

METHODS AND MEANINGS

Forms of a Quadratic Function

There are three main forms of a quadratic function: standard form, factored form, and graphing form. Study the examples below. Assume that $a \neq 0$ and that the meaning of a, b, and c are different for each form below.

Standard form: $f(x) = ax^2 + bx + c$. The y-intercept is $(0, c)$.

Factored form: $f(x) = a(x+b)(x+c)$. The x-intercepts are $(-b, 0)$ and $(-c, 0)$.

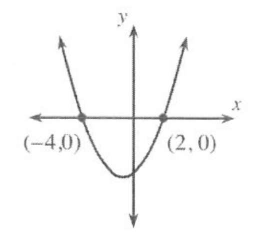
Graphing form (vertex form): $f(x) = a(x-h)^2 + k$. The vertex is (h, k).

Review & Preview

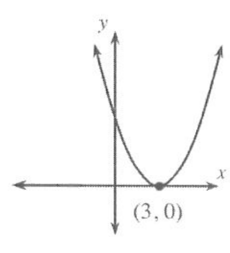
8-92. QUALITY CONTROL, Part Two

Lots O'Dough, a wealthy customer, would like to order a variety of parabolas. However, he is feeling pressed for time and said that he will pay you *lots* of extra money if you complete his order for him. Of course you agreed! He sent you sketches of each parabola that he would like to receive. Determine a possible equation for each parabola so that you can pass this information on to the Manufacturing Department.

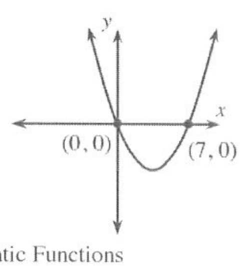
a.



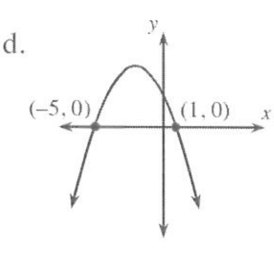
b.



c.

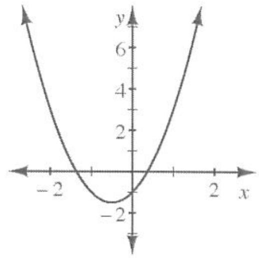


d.



8-93. Find the slope and y-intercept of the graph of $6y - 3x = 24$.

8-94. Examine the graph of $y = 2x^2 + 2x - 1$ at right.



a. Estimate the zeros of $2x^2 + 2x - 1$ from the graph.

b. What happens if you try to use the Zero Product Property to find the roots of $2x^2 + 2x - 1 = 0$?

8-95. Solve the equations below for x. Check your solutions.

a. $x^2 + 6x - 40 = 0$

b. $2x^2 + 13x - 24 = 0$

8-96. Calculate the value of the expressions below. Then compare your answers from parts (a) and (b) to those in part (a) of problem 8-95, and parts (c) and (d) to part (b) in problem 8-95. What do you notice?

a. $\frac{-6+\sqrt{6^2-(4)(1)(-40)}}{2\cdot 1}$

b. $\frac{-6-\sqrt{6^2-(4)(1)(-40)}}{2\cdot 1}$

c. $\frac{-13-\sqrt{13^2-(4)(2)(-24)}}{2\cdot 2}$

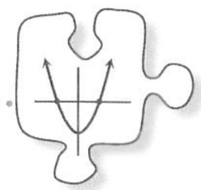
d. $\frac{-13+\sqrt{13^2-(4)(2)(-24)}}{2\cdot 2}$

8-97. Use any method to solve the systems of equations below.

a. $2x - 3y = 5$
$4x + y = 3$

b. $m = -3 + 2n$
$4m + 6n = -5$

8.2.5 How can I write it in graphing form?

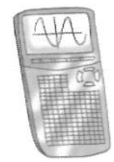


Completing the Square

In Lesson 8.2.3 you found that writing the equation of a parabola in graphing form, $f(x)=a(x-h)^2+k$, made it easier to find the vertex and the x-intercept. But how can you change standard form, $f(x)=ax^2+bx+c$, into graphing form? In this lesson, you will learn a new method called "completing the square."

8-98. Use the Zero Product Property to find the exact vertex of $y=x^2+8x+10$. What went wrong? How could you work around this problem?

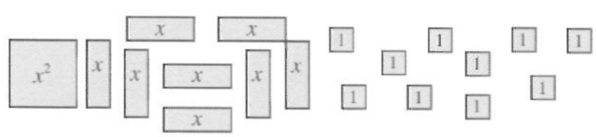
8-99. The function from problem 8-98 can be rewritten in graphing form as $y=(x+4)^2-6$. Find the exact x-intercepts and vertex. Make a sketch, then check your sketch with your graphing calculator.



8-100. COMPLETING THE SQUARE

Jessica was at home struggling with her Algebra homework. She had missed class and did not understand the new method called **completing the square**. She was supposed to use it to change $y=x^2+8x+10$ to graphing form. Then her precocious younger sister, who was playing with algebra tiles, said, "*Hey, I bet I know what they mean.*" Anita's Algebra class had been using tiles to multiply and factor binomials.

Anita explained: "$x^2+8x+10$ *would look like this;*"



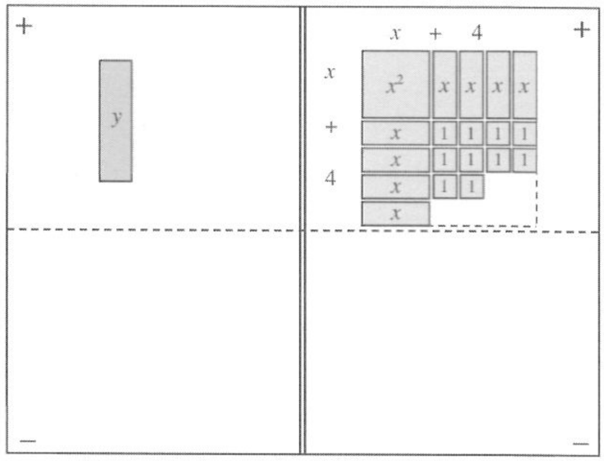
"Yes," said Jessica, *"I'm taking Algebra too, remember?"*

Anita continued, *"And you need to make it into a square!"*

Problem continues on next page. →

8-100. *Problem continued from previous page.*

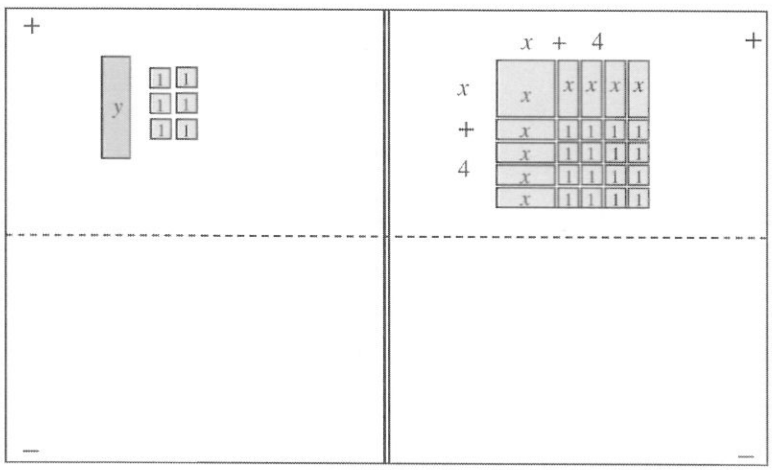
"OK," said Jessica, and she arranged her tiles on an equation mat as shown at right.



"Oh," said Jessica. *"I need 16 small unit tiles to fill in the corner!"*

"But you only have 10," Anita reminded her.

"Right, I only have ten," Jessica replied. She drew the outline of the whole square and said: *"Oh, I get it! To* ***complete the square****, I need to add six tiles to each side of the equation:"*



"Oh, I see," said Anita. *"You started with* $y = x^2 + 8x + 10$*, but now you can rewrite it as* $y + 6 = (x+4)^2$*."*

"Thank you so much, Anita! Now I can easily write the function in graphing form, $y = (x+4)^2 - 6$*."*

How can you use your graphing calculator to verify that $y = x^2 + 8x + 10$ and $y = (x+4)^2 - 6$ are equivalent functions?

8-101. Write each function in graphing form, then state the vertex and y-intercept of each parabola.

a. $f(x)=x^2+6x+7$

b. $f(x)=x^2+4x+11$

8-102. Help Jessica with a new problem. She needs to complete the square to write $y=x^2+4x+9$ in graphing form.

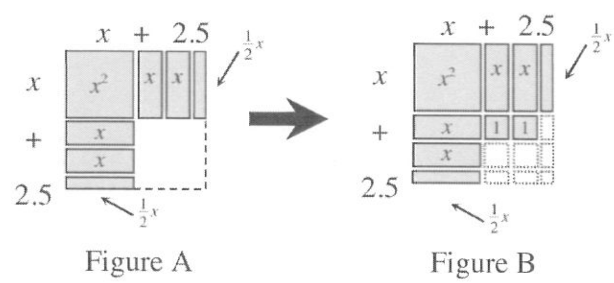
a. Draw tiles to help her figure out how to make this expression into a square. Does she have too few or too many unit squares this time? Write her function in graphing form.

b. Find the vertex and the x-intercepts. What happened? What does that mean?

c. Algebraically find the y-intercept. Sketch the graph.

8-103. How could you complete the square to change $f(x)=x^2+5x+2$ into graphing form? How would you split the five x-tiles into two equal parts?

Jessica decided to use force! She cut one tile in half, as shown below. Then she added her two small unit tiles.



a. How many small unit tiles are missing from Jessica's square?

b. Write the graphing form of the function, name the vertex and y-intercept, and sketch the graph.

8-104. Write each function in graphing form, then state the vertex and y-intercept of each parabola.

a. $f(x)=x^2+10x$

b. $f(x)=x^2+7x+2$

8-105. GENERALIZATION CHALLENGE

Work with your team to write the graphing form for $f(x)=x^2+bx+c$. Be prepared to share your strategies with the class.

Review & Preview

8-106. For each quadratic function below, use the idea of completing the square to write it in graphing form. Then state the vertex of each parabola.

a. $f(x)=x^2+6x+15$

b. $y=x^2-4x+9$

c. $f(x)=x^2+8x$

d. $y=x^2+5x-2$

8-107. Use factoring and the Zero Product Property to solve each equation.

a. $(x-4)(2x+1)=0$

b. $x^2+5x+6=0$

c. $x(2x-5)=0$

d. $x^2+4x=0$

8-108. Over a four month period the price of an ounce of gold steadily increased from \$1000 to \$1400. What was the monthly multiplier? What was the monthly percent increase?

8-109. Sketch the parabola $y=2x^2+6x+4$ by using its intercepts.

8-110. For the line with equation $4(y-2)=3(x+7)$:

a. State the slope and y-intercept.

b. Is (–7, 2) a point on the line? Explain your reason.

8-111. This problem is a checkpoint for interpreting associations. It will be referred to as Checkpoint 8.

A random sample of competitive cyclists had their maximum sustainable power output (watts) versus VO_2max tested. Their data is shown in the table below. Note: VO_2max, also called "aerobic capacity," is a measure of how much oxygen your body uses when exercising at a maximal effort for an extended period of time.

VO_2max (ml/kg/min)	Power (watts)
54	292
51	362
49	280
43	293
53	280
59	413
64	358
58	293
56	342
55	335
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a. Create a model by finding the LSRL. Sketch the graph and the LSRL.

b. What power does the model predict for the cyclist that has a VO_2max of 43 ml/kg/min? Consider the precision of the measurements in the table and use an appropriate precision in your result.

c. Find the residual for the cyclist in part (b).

d. Find the correlation coefficient and interpret it.

e. Describe the association. Provide numerical values for *direction* and *strength* and interpret them in context.

Check your answers by referring to the Checkpoint 8 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 8 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

Chapter 8 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 8.1.1 – Diagonals of a Generic Rectangle
- Lesson 8.1.3 – Factoring Quadratics
- Lesson 8.1.5 – Factoring Shortcuts
- Lesson 8.2.1 – Quadratic Web
- Lesson 8.2.2 – Zero Product Property
- Lesson 8.2.3 – Strategies for Finding x-Intercepts

Math Notes

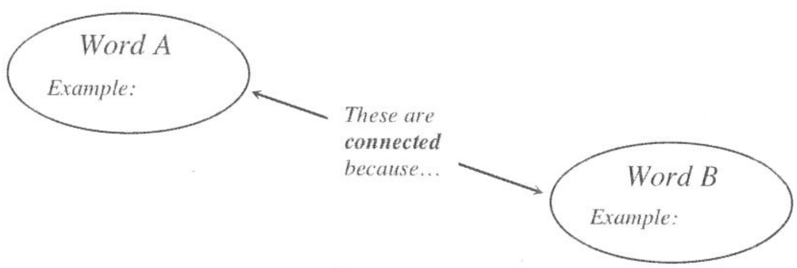
- Lesson 8.1.1 – More Vocabulary for Expressions
- Lesson 8.1.2 – Diagonals of Generic Rectangles
- Lesson 8.1.3 – Standard Form of a Quadratic Expression
- Lesson 8.1.4 – Factoring Quadratic Expressions
- Lesson 8.2.2 – Zero Product Property
- Lesson 8.2.4 – Forms of a Quadratic Function

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

binomial	factor	generic rectangle
graph	monomial	parabola
product	quadratic equation	situation
root	solution	standard form
zeros	graphing form	difference of squares
sum	symmetry	factor completely
vertex	x-intercept	table
y-intercept	Zero Product Property	perfect square trinomial
factored form	completing the square	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Carefully copy your work from problems 8-90 and 8-92, modifying and expanding it if necessary, to showcase your understanding of parabolas.

Then showcase your growth in understanding x-intercepts since Chapter 7 by answering the following questions:

How many different kinds of graphs can you create that have:
a. No x-intercepts?
b. One x-intercept?
c. Two x-intercepts?
d. Three or more x-intercepts?

Modify and expand your work from the Chapter 7 portfolio entry as needed. (Do not actually change your Chapter 7 entries, but instead make new entries here in your portfolio for Chapter 8, showing your growth in understanding x-intercepts.) For each type of graph, show a sketch, label the key points, and give its equation. Make sure that each graph you give as an example represents a different family, and describe the family in words or with a general equation. Show how to calculate the x-intercepts of each of your sample graphs.

Your teacher may give you the Chapter 8 Closure Resource Page: Quadratic Multiple Representations Graphic Organizer. Complete the Resource Page using Imp's water balloon toss from problem 8-55 and part (e) of problem 8-91. Showcase your current understanding of the multiple representations of a quadratic function.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 8-112. Factor and use the Zero Product Property to find the roots of the following quadratic equations.

a. $0 = x^2 - 7x + 12$

b. $0 = 6x^2 - 23x + 20$

c. $0 = x^2 - 9$

d. $0 = x^2 + 12x + 36$

CL 8-113. The price of milk has been steadily increasing 5% per year. If the cost of a gallon is now \$3.89:

a. What will it cost in 10 years?

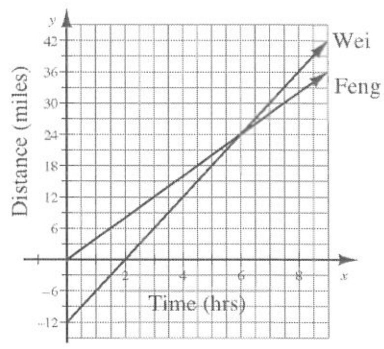
b. What did it cost 5 years ago?

CL 8-114. Use the graph at right to answer the questions below.

a. One of these lines represents Feng, and one represents Wai. Write an equation for each girl's line.

b. The two girls are riding bikes. How fast does each girl ride?

c. When do Feng and Wai meet? At that point, how far are they from school?



CL 8-115. Graph $y = x^2 - 2x$. Identify the roots, y-intercept, x-intercepts, and the vertex.

CL 8-116. Find the coordinates of the y-intercept and x-intercepts of $y = x^2 - 2x - 15$. Show all of the work that you used to find these points.

CL 8-117. Without using a calculator, simplify using only positive exponents.

a. $(9^{1/2}x^2y)(27^{1/3}y^{-1})$

b. $(x^{1/2})^{-2}$

c. $(\frac{1}{125})^{2/3}$

d. $\frac{8x^3}{-2x^{-2}}$

CL 8-118. Quinn started off with twice as much candy as Denali, but then he ate 4 pieces. When Quinn and Denali put their candy together, they now have a total of 50 pieces. How many pieces of candy did Denali start with?

CL 8-119. Given the two points $(-24,7)$ and $(30,25)$,

a. What is an equation of the line passing through the points?

b. Is $(51,33)$ also on the same line? Explain your reasoning.

CL 8-120. Write the equation of the following two sequences in "first term" form.

a. $100, 10, 1, 0.1, \ldots$

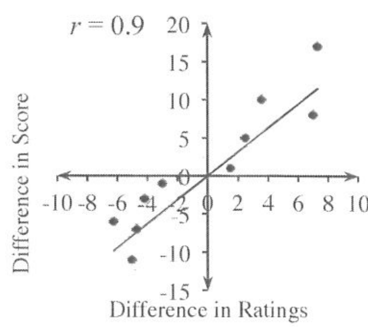
b. $0, -50, -100, \ldots$

CL 8-121. Write the first four terms of the following sequences.

a. $a_n = 3 \cdot 5^{n-1}$

b. $a_1 = 10,\ a_{n+1} = -5a_n$

CL 8-122. Bianca plays lacrosse for her high school team. She loves to look at the ratings of lacrosse teams that play in her area. Bianca looked at the difference in ratings between the two teams. She wondered if she could predict the differences in the final scores when the two teams played each other. Bianca drew the following scatterplot by hand.



a. Discuss the association, including an interpretation of slope and R-squared.

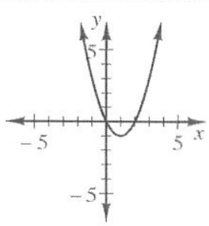
b. Predict the outcome of the game if Novato (rating 74.27) played Marin Catholic (rating 76.61).

c. The difference in rating between Woodbridge Lakeside was 5.00. When they played, their residual was 7. What was the difference in their scores in this game?

CL 8-123. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 8-112.	a. $x = 4$ or $x = 3$ b. $x = \frac{5}{2}$ or $x = \frac{4}{3}$ c. $x = -3$ or $x = 3$ d. $x = -6$	Section 8.2 MN: 8.2.2 LL: 8.2.2	Problems 8-17, 8-31, 8-39, 8-49, 8-71, 8-74, 8-83, 8-85, 8-95, and 8-107
CL 8-113.	a. \$6.34 b. \$3.05	Lesson 5.3.2 LL: 5.1.1 and 5.3.2	Problems CL 6-129 CL 7-117, 8-20, and 8-108
CL 8-114.	a. Feng: $y = 4x$ Wai: $y = 6x - 12$ b. Feng rides at 4 miles per hour; Wai rides at 6 miles per hour. c. Feng and Wai meet after 6 hours. At that point, they are 24 miles from school.	Chapter 2 MN: 2.2.2 LL: 2.2.2	Problem CL 2-101
CL 8-115.	roots: 0 and 2 y-intercept: $(0, 0)$ x-intercepts: $(0, 0)$ and $(2, 0)$ vertex: $(1, -1)$ 	Lesson 1.1.3 and Section 8.2	Problems CL 2-108, 8-11, 8-44, 8-58, 8-69, 8-83, 8-94, and 8-109
CL 8-116.	y-intercept: -15 x-intercepts: 5 and -3	Section 8.2 LL: 8.2.3	Problems 8-42, 8-73, 8-83, and 8-109
CL 8-117.	a. $9x^2$ b. $\frac{1}{x}$ c. $\frac{1}{25}$ d. $-4x^5$	Lesson 7.2.1 MN: 7.2.2 LL: 7.2.1	Problems CL 7-115, 8-32, 8-50, and 8-75

Problem	Solution	Need Help?	More Practice
CL 8-118.	Denali has 18 pieces of candy.	Chapter 4 Checkpoints 7A and 7B MN: 4.2.3 LL: 4.2.3	Problems CL 7-118, 8-29, and 8-41
CL 8-119.	a. $y=\frac{1}{3}x+15$ b. No, it does not work in the equation.	Lessons 2.1.4 and 2.3.1 Checkpoint 5B LL: 2.3.1	Problems CL 2-109, CL 3-119, CL 5-129, CL 7-121, and 8-23
CL 8-120.	a. $t(n)=100\cdot 0.1^{n-1}$ b. $t(n)=0-50(n-1)=-50(n-1)$	Sections 5.2 and 5.3	Problems 7-108, 8-8, 8-19, 8-40, and 8-52
CL 8-121.	a. $3, 15, 75, 375$ b. $10, -50, 250, -1250$	Sections 5.2 and 5.3	Problems CL 5-125 and CL 5-130
CL 8-122.	a. There is a very strong positive linear association between the differences in ratings and the corresponding difference of game scores. There are no apparent outliers. About 81% of the variation in score differences is explained by the difference in ratings of the two teams. The slope is about 1.6. An increase of 1 in the difference of ratings is expected to increase the score difference by 1.6 points. b. The LSRL is about $y=1.6x$. $y=1.6(74.27-76.61)=-3.74$. Marin Catholic is expected to win by about 4 goals. c. The model predicts the difference in scores to be $y=1.6(5)=8$ points. Since residual = actual – predicted, 7 = actual =– 8. The actual difference in scores was 15 points.	Chapter 6 Checkpoint 8	Problems CL 6-122, CL 6-123, CL 6-124, CL 7-122, 8-34, 8-54, 8-61, and 8-111